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The influence of tie strength on evolutionary games on networks: An empirical investigation

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ABSTRACT

Extending previous work on unweighted networks, we present here a systematic numerical investigation of standard evolutionary games on weighted networks. In the absence of any reliable model for generating weighted social networks, we attribute weights to links in a few ways supported by empirical data ranging from totally uncorrelated to weighted bipartite networks. The results of the extensive simulation work on standard complex network models show that, except in a case that does not seem to be common in social networks, taking the tie strength into account does not change in a radical manner the long-run steady-state behavior of the studied games. Besides model networks, we also included a real-life case drawn from a coauthorship network. In this case also, taking the weights into account only changes the results slightly with respect to the raw unweighted graph, although to draw more reliable conclusions on real social networks many more cases should be studied as these weighted networks become available.

1. Introduction

The importance of the population structure on evolutionary game theory has been fully realized in recent years. In fact, the customary infinite well-mixed populations used in the theory have the appeal of simplicity and lend themselves to exact mathematical analysis [1] but network science has clearly shown that fully mixed populations are only an approximation, sometimes a bad one, to the actual interactions among agents. These social interactions can instead be more precisely represented as graphs in which nodes represent agents and links stand for their relationships [2]. In the last few years evolutionary games on networks have been thoroughly investigated and many results are available. Most of them come from numerical simulations, but there are also some theoretical results, mainly on degree-homogeneous graphs. It would be impossible to cite all the works in this fast-developing field but good recent reviews can be found in [3–5].

The bulk of the work on evolutionary games on complex networks so far has dealt with unweighted graphs, so that the intensity of the relationships has not been taken into account in general. This is right as a first step and allows one to ignore the interplay between topology and structure, but a further step toward more realism consists in including the strength of a relationship. Indeed, there are only few works in which the role of link weights in evolutionary game dynamics have been considered [6,7]. However, such investigations have been rather limited in character and there is not, as far as we know, any systematic treatment dealing with this potentially important aspect of games on networks. In the present work we present such a study of the behavior of paradigmatic evolutionary games on weighted networks.

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Table 1
Generic payoff bi-matrix for the two-person, two-strategies symmetric games discussed in the text.

	C	D
C	(R, R)	(S, T)
D	(T, S)	(P, P)

Attributing weights to links in technological networks such as computer networks, power grids, or airline route networks, is relatively easy. For example, data packets for a computer network, and number of flights, passengers, or seats for airlines are quantities that make sense and can be defined and measured easily and without ambiguity. Contrastingly, attributing weights to relationships in social networks is not a simple matter because the relationship is often multi-faceted and relies on psychological and sociological features that are difficult to define and measure, such as friendship, empathy, and common beliefs. Nevertheless, there are some social networks for which at least a proxy for the intensity of a relationship can be defined and accurately measured. This is the case, for instance, for e-mail networks, phone calls networks, and coauthorship networks among others [8–10]. Other examples come from the field of animal networks in which animals can be marked or recognized in some way and repeated co-occurrences of animals adds to the weight of their relationship [11].

For the sake of definiteness, we shall assume in the following that suitable weights can be attributed to the network edges and, since we shall explore several possible ways of performing the assignment, our work will be primarily based on standard model networks in order to try to unravel the interplay and the correlation between the purely topological aspect of the relationships and their intensity for the chosen games. Nevertheless, to connect our work to real-life nets, we shall also consider a known collaboration network. Moreover, to avoid having to deal with further degrees of freedom, we use fixed networks, i.e. networks in which neither the number of nodes nor the number of links is subject to change over time. Likewise, there is no rewiring of existing links among the network nodes. Co-evolutionary models can be more realistic (see e.g. [12–14,5]) but in this work we are especially interested in singling out the effect of link weights in static networks, which is a satisfactory approximation if the network dynamics is slowly fluctuating. The present investigation is based on extensive numerical simulations.

The article is organized as follows. In the next section we briefly introduce the main games used and their parameter space, as well as the evolutionary rules for strategy update. In Section 3 we provide justifications for a number of ways of attributing weights to network links and we numerically study evolutionary games behavior using typical model networks and several link weight distributions. Finally, we discuss the results and give our conclusions in Section 4.

2. Evolutionary games on networks

2.1. The standard games

We study the four standard two-person, two-strategy, symmetric games, namely the Prisoner's Dilemma (PD), the Snowdrift Game (SG), the Stag Hunt (SH), and the Harmony game (HG). We briefly summarize the main features of these games here for completeness; more detailed accounts can be found elsewhere [1,15]. The games have the generic payoff bi-matrix of Table 1. In this matrix, R stands for the *reward* the two players receive if they both cooperate (C), P is the *punishment* for bilateral defection (D), and T is the *temptation*, i.e. the payoff that a player receives if she defects while the other cooperates. In this case, the cooperator gets the *sucker's payoff* S. In order to study the standard parameter space, we restrict the payoff values in the following way: $R = 1$, $P = 0$, $-1 < S < 1$, and $0 < T < 2$. In the resulting *TS*-plane, each game corresponds to a different quadrant depending on the ordering of the payoffs.

For the PD, the payoff values are ordered such that $T > R > P > S$. Defection is always the best rational individual choice, so that (D, D) is the unique Nash Equilibrium (NE) and also the only Evolutionarily Stable Strategy (ESS) [1]. Mutual cooperation would be socially preferable but C is strongly dominated by D.

In the Snowdrift game, the order of P and S is reversed, yielding $T > R > S > P$. Thus, in the SD when both players defect they each get the lowest payoff. (C, D) and (D, C) are NE of the game in pure strategies. There is a third equilibrium in mixed strategies where strategy D is played with probability p, and strategy C with probability $1 - p$, where p depends on the actual payoff values. The only ESS of the game is the mixed strategy, while the two pure NE are not ESSs [1]. Players have a strong incentive to play D, which is harmful for both parties if the outcome produced happens to be (D, D).

In the Stag Hunt, the ordering is $R > T > P > S$, which means that mutual cooperation (C, C) is the best outcome, Pareto-superior, and a NE. The second NE, where both players defect is less efficient but also less risky. The dilemma is represented by the fact that the socially preferable coordinated equilibrium (C, C) might be missed for "fear" that the other player will play D instead. The third mixed-strategy NE in the game is evolutionary unstable and not an ESS [1].

Finally, in the Harmony game $R > S > P > T$. In this case C strongly dominates D and the trivial unique NE is (C, C). The HG is non-conflicting by definition and does not cause any dilemma: we include it just to complete the quadrants of the parameter space.

2.2. Population structure and dynamics

We represent the population of players by an undirected weighted graph $G(V, E)$, where the set of vertices V represents the individuals, and the set of weighted edges E represents their symmetric interactions. The weight of an edge $e \in E$ will be denoted by w_e or by w_{ij}, by using the edge end points i and j and the weights w_e are normalized in $[0, 1]$. The population size N is the cardinality of V. A neighbor of an agent i is any other agent j adjacent to i. The set of neighbors of i is denoted by V_i. The cardinality of this set is the degree k_i of vertex $i \in V$; $p(k)$ denotes the degree distribution of the graph, i.e. the probability that an arbitrarily chosen node has degree k. The average degree of the network is given by $\langle k \rangle = \sum_k k\,p(k)$. For the weighted aspects of the network $p(w_e)$ represents the link weight distribution, and $p(s)$ denotes the *strength* distribution, where the strength $s(i)$ of a vertex i is defined as $s(i) = \sum_{j \in V_i} w_{ij}$, i.e. the sum of the weights of the links incident in i [16].

For the evolutionary dynamics, we must next define the decision rule by which individuals update their strategy and the timing of the dynamical process. There are several possible strategy update rules that can be used [3,4]. The results are not very qualitatively dependent on the specific rule type, although there are quantitative differences between them [4]. For the sake of simplicity, we use two rules that are sufficiently different so as to represent typical diverse behavior: *imitation of the best* and *local replicator dynamics*. These updating rules will be explained below.

Let $\sigma_i \in \{C, D\}$ be the current strategy of player i and let us call M the payoff matrix of the game. The quantity

$$\Pi_i(t) = \sum_{j \in V_i} \sigma_i(t)\; M\; \sigma_j^T(t)$$

is the accumulated payoff collected by player i at time step t. Since we work with weighted networks, the pairwise payoffs $M_{ij} = \sigma_i\; M\; \sigma_j^T$ are multiplied by the weights w_{ij} of the corresponding links before computing the accumulated payoff earned by i. This takes into account the relative importance of the relationship as represented by its weight.

Another possibility is to use the normalized weight w_{ij} as a probability of interaction of agents i and j, i.e. i and j will play with probability w_{ij} as in Ref. [7]. Here we use the former choice but some numerical experiments have shown that the results do not differ qualitatively by using the latter instead.

In imitation of the best, the strategy $\sigma_i(t)$ of individual i at time step t will be

$$\sigma_i(t) = \sigma_j(t-1),$$

where

$$j \in \{V_i \cup i\} \quad s.t.\, \Pi_j = \max\{\Pi_k(t-1)\}, \quad \forall k \in \{V_i \cup i\}.$$

That is, individual i will adopt the strategy of the player with the highest payoff among its neighbors including itself. If there is a tie, the winner individual is chosen uniformly at random, but otherwise the rule is deterministic.

The local replicator dynamics rule is stochastic and it is consistent with the original mean-field evolutionary game theory equations [17]. Here it has been slightly modified to take into account the weighted nature of the network. Player i's strategy σ_i is updated by drawing another player j from the neighborhood V_i with a probability proportional to w_{ij}, and replacing σ_i by σ_j with probability

$$p(\sigma_i \to \sigma_j) = (\Pi_j - \Pi_i)/K,$$

if $\Pi_j > \Pi_i$, and keeping the same strategy if $\Pi_j \leq \Pi_i$. $K = \max(s_i, s_j)[\max(1, T) - \min(0, S)]$, with s_i and s_j being the strengths of nodes i and j respectively, ensures proper normalization of the probability $p(\sigma_i \to \sigma_j)$.

Finally, if we define $C(t) = (\sigma_1(t), \ldots, \sigma_N(t))$ as being the configuration or state of the population at time t, then the simulation advances synchronously according to the symbolic global evolution rule F:

$$C(t+1) = F(C(t)).$$

In other words, all the individuals in the network play the game once with all their respective neighbors, accumulate their payoffs, and decide their strategy for the next time step according to the above rules. The evolution could also be fully or partially asynchronous. In partially asynchronous dynamics a fraction f of the population is simultaneously updated in each time step [14]. In fully asynchronous update an individual is chosen at random, she plays the game once with her neighbors, and she updates her strategy accordingly. Then the payoff of all individuals is set to zero and another individual is drawn uniformly at random in the whole population at the next time step. In several studies, following an enquiry by Huberman and Glance [18], it has been shown that asynchronous evolution doesn't change the main qualitative aspects of the dynamics of games on networks (for example, see Refs. [17,4,14]). Thus, here we use synchronous dynamics.

2.3. Simulation parameters

All simulations were performed for a networked population size of $N = 2000$ and mean degree $\langle k \rangle = 8$ unless otherwise stated. The initial density of cooperators is 0.5, uniformly distributed over the vertices of the networks. Given that our main goal here is to compare weighted and unweighted networks with respect to evolutionary games, in the interest of simplicity we do not explore unbalanced initial conditions. Each value in the phase space reported in the following figures is the average of 50 independent runs. Each run has been performed on a fresh realization of the corresponding graph. To detect steady states[1] of the dynamics we first let the system evolve for a transient period of $5000 \times N$ time steps. After a quasi-equilibrium state is reached past the transient, averages are calculated during $500 \times N$ additional time steps. A steady state has always been reached in all simulations performed within the prescribed amount of time, for most of them well before the limit. The state space explored is defined by $R = 1$, $P = 0$, $-1 < S < 1$, and $0 < T < 2$ and the T and S axes have been sampled at intervals of 0.1.

3. Games on weighted networks

In the last decade the structure of hundreds of medium to large networks have been investigated thanks to readily available electronic data [2]. However, the large majority of these graphs were of the unweighted type. The reasons are that, as hinted at in the Introduction, apart from technological or economic networks such as trade networks, in the realm of social nets it is often difficult to associate sensible weights to an edge representing some kind of relationship between two agents. However, there are some published studies that can be used as a starting point to estimate suitable forms for the weights, given that some very different hypothesis have been voiced in the literature. One extreme position, called the *dyadic hypothesis* [9] is to argue that the weight of a particular tie does not depend on the network structure around the two concerned agents but only on the nature of their relationship. In this view, tie strengths are completely uncorrelated with topological features such as node degree and clustering coefficient. In two detailed studies of large social networks, the first being a scientific coauthorship network [19] and the second a mobile call [8] network, it has been empirically shown that this is not the case. Although the non-correlation hypothesis does not seem to be a likely one in social networks, it is still useful as a benchmark: a kind of null model against which to test some more realistic assumptions. In our first simulation model we thus assume that weights are attributed to links without correlation with the topology. In order to get rid of topological effects and to observe the effect of the link strength only on the dynamics, we first model the games on regular random graphs in which each node has the same degree but links are otherwise randomly distributed. Random graphs are the closest network approximation to a mean-field well mixed population, ideally represented by a complete network and, in the limit of very large population sizes, the standard results of evolutionary game theory should hold [1,4] at least approximately.

Fig. 1 depicts the average cooperation levels on regular random graphs of degree $k = 8$ and $N = 2000$ nodes at steady state, when the weights are assigned at random according to a uniform distribution in $[0, 1]$ distributed uniformly at random among the available links.

The images on the left correspond to imitate the best update rule while on the right they correspond to local replicator dynamics. Unsurprisingly, the results are almost identical to those obtained on the same family of graphs but using unweighted networks (bottom row images). This result is also fully coherent with the cooperation levels found in Ref. [4] for unweighted Erdös–Rényi random graphs with the same $\langle k \rangle$. Clearly, when weights are assigned uniformly at random there is an implicit averaging over the whole set of edges when calculating payoffs and the inclusion of weights does not not change the qualitative results. In passing, we note the remarkable level of cooperation reached in the SH game which imitates the best strategy update rule, a phenomenon already observed in Ref. [4].

To investigate whether degree inhomogeneity changes the picture, we have also simulated the same games on Barabási–Albert (BA) scale-free graphs of the same size and $\langle k \rangle = 8$. The results, shown in Fig. 2 (top row) are again very similar to the unweighted cases (bottom row), and there is full agreement with the results of Roca et al. [4]. One is thus led to the conclusion that, when weights are distributed uniformly at random among the edges there is almost no difference with the unweighted case for both update rules. This in turn shows that when weights are totally uncorrelated with the topological aspects of the network, such as degree or clustering coefficient, their influence is negligible.

Indeed, even when the weight distribution is a long-tailed one such as a power-law $p(w_e) \propto w_e^{-\gamma}$, the results are very similar, as shown in Fig. 3, where the value of the exponent γ is 2. The reasons seem intuitively clear: since there are few strong links, hubs with many connections will get only a few of those, which will change the picture very little with respect to the unweighted case. Likewise, the few strong links that will exist among poorly connected vertices, because of the low degree of the end points, will not be able to influence a sizable portion of the network. Only statistical outliers could change this significantly but, over many graph realizations, the fluctuations will be smoothed and only mean values will matter.

The inescapable conclusion is the following: if one assumes the dyadic hypothesis for setting the edge weights, owing to system averaging, there is almost no effect on cooperation. But we have already remarked that empirical research to date indicates that edge weights and topological properties are related. To take weight–degree correlations into account one possible approach is to assume that $w_{ij} \propto (k_i k_j)^\alpha$ for some small exponent α. Such an empirical correlation has indeed been detected for the world-wide airport network [19] with $\alpha \approx 1.5$, and similar behavior, perhaps with different values of the exponent, seems to be likely in all kind of transportation networks in which there are fluxes that must respect local conservation [9]. However, social networks are different in this respect, they are much more local and there aren't any

[1] True equilibrium states in the sense of dynamical systems stability are not guaranteed to be reached by the simulated dynamics. For this reason we prefer to use the terms steady states or quasi-equilibrium states which are states that have little or no fluctuation over an extended period of time.

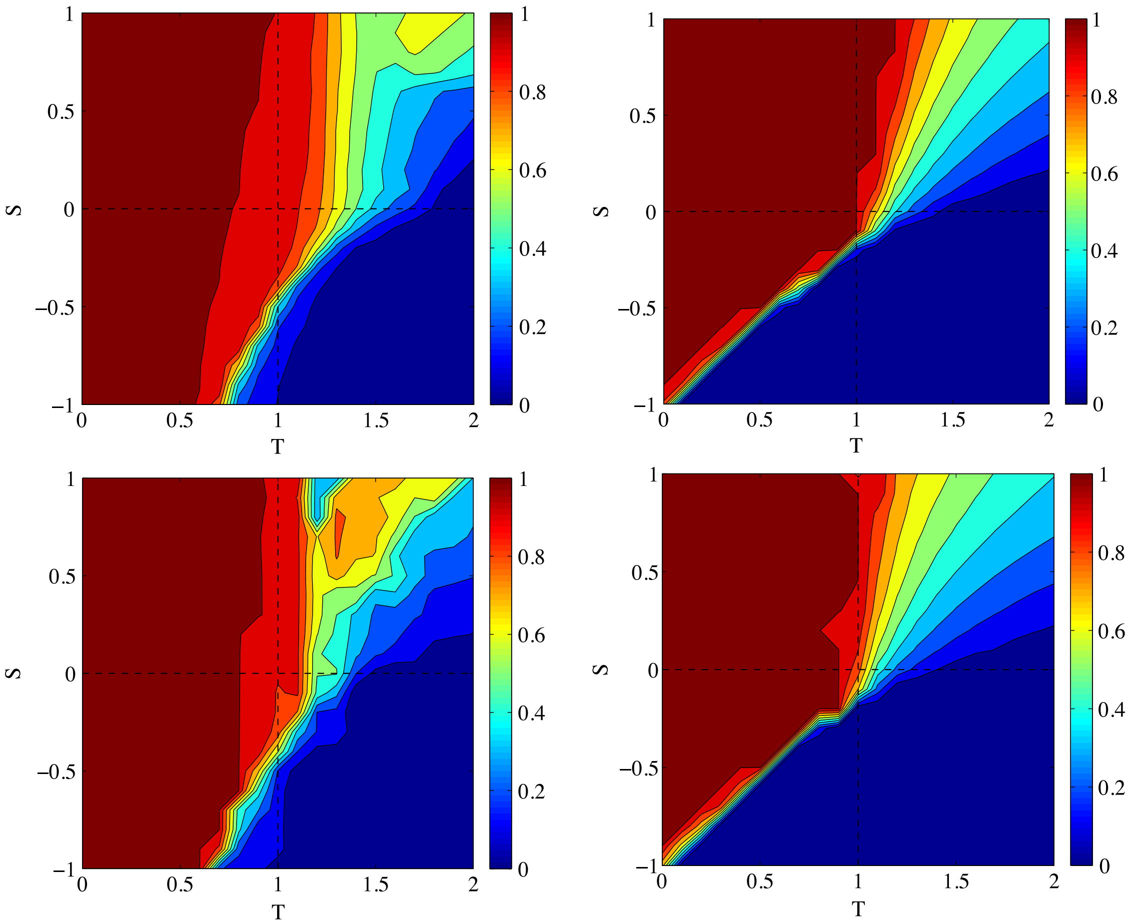

Fig. 1. Average degree of cooperation at steady state for regular random graphs with $\langle k \rangle = 8$. Left column: imitation of the best. Right column: replicator dynamics. Top row: link weights uniformly and independently distributed. Bottom row: unweighted regular random graphs. Network sizes $N = 2000$. Each grid point value is the average over 50 independent runs. Blue means more defection.

obvious quantities that could constrain the relationship between link weights and number of contacts. For example, both [19,8,9] found that $\langle w_{ij} \rangle$ is uncorrelated with $k_i \times k_j$ for mobile phone call nets as well as for a coauthorship network.

In Ref. [6] Du et al. have tried to account for the effect of link weights on the PD on Barabási–Albert scale-free graphs with the Fermi function [3]:

$$p(\sigma_i \to \sigma_j) = \frac{1}{1 + \exp(-\beta(\Pi_j - \Pi_i))},$$

as a strategy update rule with $\beta = 10$. This setting gives results very similar to our replicator rule in unweighted networks, as it has been clearly shown in Ref. [4]. The payoff was rescaled using the corresponding link weight as explained in Section 2.2. They assumed the above degree product form for the weights, studying the evolutionary behavior of the PD for several negative and positive values of the exponent α. However, their simulations only covered a tiny part of the game phase space due to their use of the so-called "reduced" PD game in which $R = 1$ and $S = P = 0$, which makes T the only free parameter and corresponds to the straight line at the frontier between the PD and the SD games. Although, as remarked above, this form of degree–weight correlation is not supported by empirical data in social networks, for the sake of completeness we performed simulations for the whole four games phase space. The results for replicator dynamics are shown in Fig. 4 where we report the average cooperation values for values of α between -3 and 3 starting with $\alpha = -3$ in the leftmost top row picture; α then increases from left to right and takes positive values starting from the second picture in the bottom row. The last top row image and the first bottom raw image correspond to the case $\alpha = 0$, i.e. the unweighted networks. From these images it appears that cooperation seems to increase around $\alpha = 1$ but it is difficult to really see it. In order to better quantify the effect, in Fig. 5 we plot the average cooperation values for each game as a function of α. Now it becomes clear that, taking the average over the whole game phase space, in the three non-trivial games there is a "plateau" of cooperation between $\alpha = 0.5$ and $\alpha = 1$ approximately (of course the HG case is only shown for completeness). For values lower than 0 and beyond 1.5 the trend is toward a lower, almost constant level of cooperation. This is confirmed by the values obtained for $\alpha = -10$ and $\alpha = 10$ which are shown for reference as small traits on each curve on the left and the right of the figure

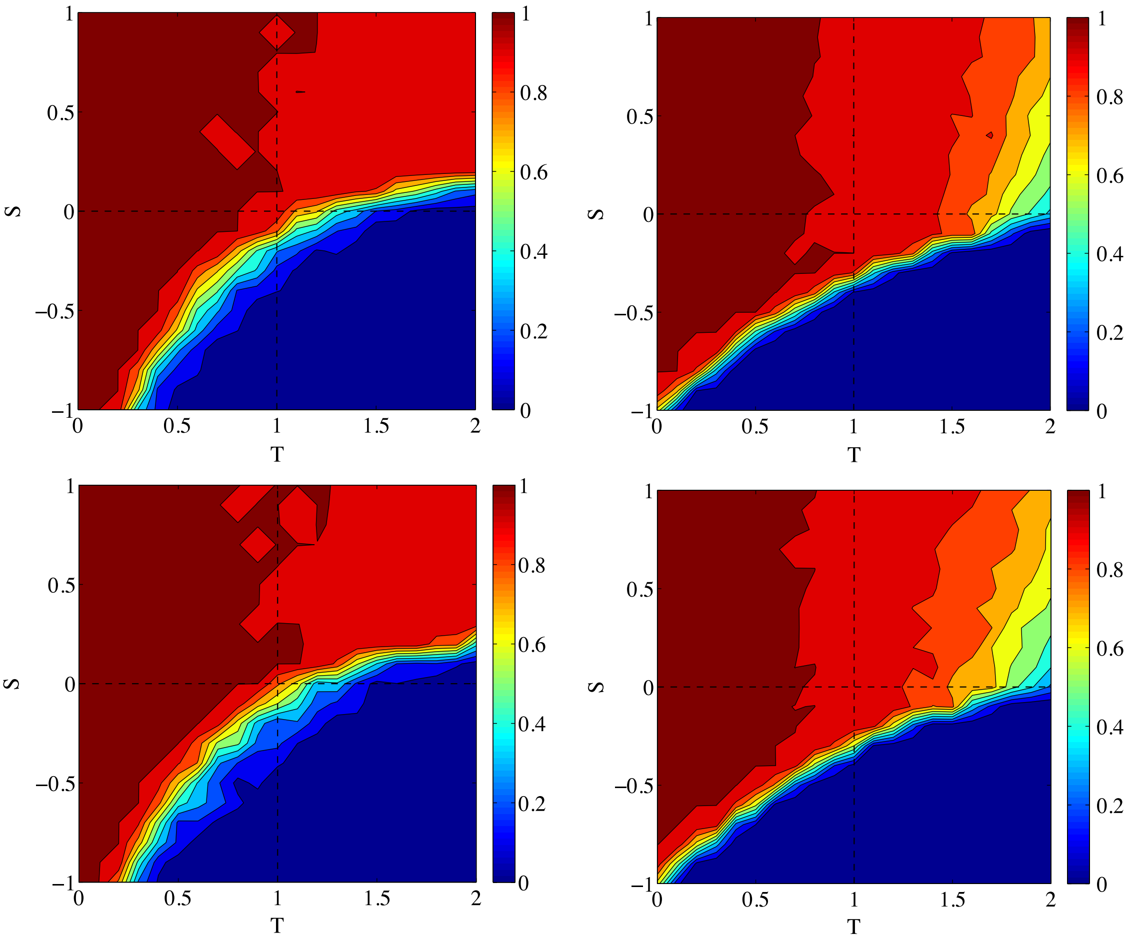

Fig. 2. Average degree of cooperation at steady state for BA scale-free networks with $\langle k \rangle = 8$. Left images: imitation of the best. Right images: replicator dynamics. Top row: link weights drawn from a uniform distribution. Bottom row: unweighted BA scale-free networks. Network sizes $N = 2000$. Each grid point value is the average over 50 independent runs. Blue stands for more defection.

respectively. Du et al. [6] found a big increase in cooperation for large negative values of α and for α close to -1 while they found a deep minimum of cooperation around $\alpha = -1.5$. This, however, only applies to the region of the space represented by the segment at the frontier between the PD and SD games. Our results show that this non-monotonic behavior is not observed when the entire phase space is taken into account.

3.1. Weighted networks from bipartite graphs

We have seen above that there can be a generally positive influence on cooperative strategies in weighted networks when the link weights are proportional to the products of the endpoints degrees with an exponent between 0.5 and 1. However, the few available empirical studies exclude the presence of such a correlation in typical social networks [19,8,9]. But many social networks are of the *affiliation* type, meaning the participation of a set of actors in a set of groups or interest centers. Each set is represented by the vertices of a graph and there is a link $X - G$ between two elements of the sets when an actor X participates to group G. In this model there can be no links between vertices belonging to the same set. Such a situation can be described by using *bipartite graphs*. Although social networks such as friendship or mutual communication nets are not of this kind, there are many significant examples of bipartite graphs in society such as scientist coauthoring an article, directors belonging to the same board, people that have bought the same book in Amazon, actors starring in the same movie, and so on [2].

A graph $G(V, E)$ in which $V = \{v_1, \ldots, v_N\}$ is the set of vertices or nodes, and $E = \{e_1, \ldots, e_M\}$ is the set of edges or links, is said to be bipartite when the vertices can be partitioned into two disjoint sets $V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$, such that there are no edges $e = \{u, v\}$ between vertices belonging to different sets:

$$\{\{u, v\} : u \in V_1, v \in V_2\}, \quad \forall e \in E.$$

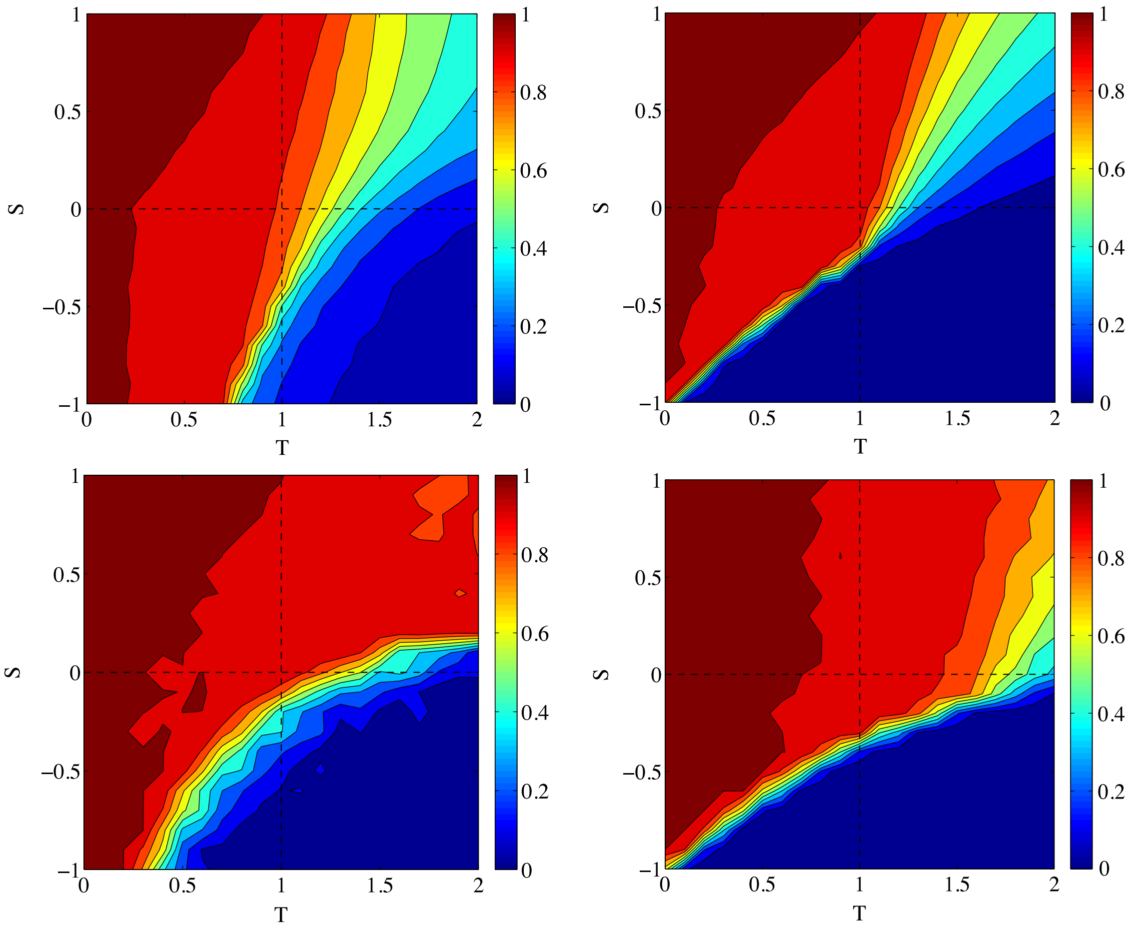

Fig. 3. Average degree of cooperation at steady state when link weights are distributed according to an inverse power-law with an exponent of 2. Left column: imitate the best. Right column: replicator dynamics. Top row: regular random graphs; bottom row: BA scale-free graphs. $N = 2000$, averages over 50 runs.

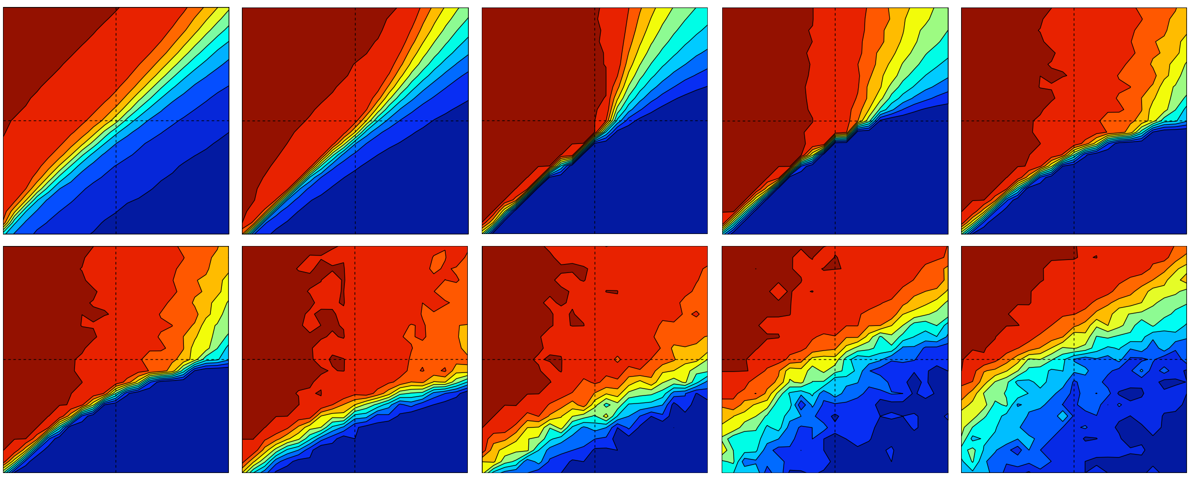

Fig. 4. Average cooperation at steady state using replicator dynamics on BA networks of size $N = 2000$ and $\langle k \rangle = 8$ as a function of the parameter α (see text). Top row images, from left to right $\alpha = -3, -2, -1, -0.5, 0$. Bottom row, from left to right $\alpha = 0, 0.5, 1, 2, 3$. The initial density of cooperators is 0.5 in all cases. Averages over 50 runs.

The *incidence matrix B* of a bipartite network with, say, l groups and m actors is an $l \times m$ rectangular matrix such that the generic matrix element B_{ij} is 1 if actor j belongs to group i and 0 otherwise [2].

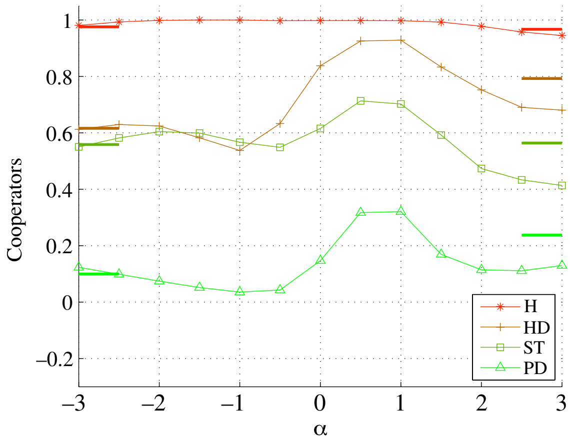

Fig. 5. The points show, for each game, the average amount of cooperation at steady state in the whole game's phase space as a function of α. Lines are just a visual guide. In the legend, PD stands for Prisoner's Dilemma; HD stands for the Hawk-Dove or Snowdrift game; ST stands for Stag Hunt, and H designates the Harmony game.

From the bipartite graph, it is an easy matter to obtain two derived graphs which are called *projections*. One can construct a graph in which two actors are connected if they adhere to the same group, or we can also build the projection in which two groups are connected if they share a common actor. The two projections capture the essence of the relationships we are looking for but they do not account for the "weight" of a relationship. Indeed, it is sensible to say that it is not the same whether two people sit together on a single board or on several, or whether an article has only two coauthors or ten. In some sense, their degree of interaction should be higher in the former case. To account for this, the projection can be weighted; for example, for the actors projection, an edge, i.e. a pair of connected actors, will have a weight equal to the number of common groups. The weighted projection can be obtained from the incidence matrix B as follows [2]:

$$P = B^T B, \quad \text{where } P_{ij} = \sum_{k=1}^{l} B^T_{ik} B_{kj} \tag{1}$$

where B^T is the transpose of B, and l is the number of groups. The elements P_{ij} of the $m \times m$ matrix P are the weights, i.e. the number of common groups shared by the actors i and j, whereas the diagonal elements P_{ii} are the number of groups to which actor i belongs. This provides us with a "natural" way of attributing ways to the links of the projection graph and thus can in principle be used to gauge the behavior of the standard games on the resulting weighted networks.

Among several existing models of bipartite graphs, we choose the team assembly model by Guimerà et al. [20]. In this growing model, teams are formed sequentially taking their members both from a set of newcomers and a set of incumbents. Teams correspond to top nodes, newcomers to new bottom nodes and incumbents to existing bottom nodes. The model starts at time zero with an endless pool of newcomers. Once they are selected for a team, newcomers become incumbents. Each time step t, a new team is formed and added to the network. The team consists of m agents. With a probability p, the agent is drawn from the pool of incumbents and with probability $1 - p$ from the pool of newcomers. If the new agent is an incumbent and there is already another incumbent in the team, the new agent is selected with probability q from the set of collaborators of a randomly selected incumbent in the team. With probability $1 - q$, it is randomly selected from the set of all incumbents.

For our graphs, we used values of $p = 0.6$, $q = 0.9$ (both empirically justified [20]) and $m = 4$. In order to generate graphs with exactly $N = 1000$ agents, we repeated the procedure described before for a number M of teams equal to $\lfloor n/m(1-p) \rfloor$ and kept only those graphs with exactly $N = 1000$ agents.

Once the graphs are constructed, we let the game dynamics develop as previously explained. The results are depicted in Fig. 6 for the unweighted case, and in Fig. 7 for the weighted graphs both for replicator dynamics and imitate the best strategy update rules. It appears that the level of cooperation is very similar for weighted and unweighted networks in both cases, with the only difference that, in the weighted case, the transition region between cooperation and defection becomes less crisp. This can be due to the fact that weights act as a form of noise in the evaluation of payoffs, which gives more fluctuations in the transition region. However, the bottom line is that, once more, the effect of the link strength on cooperation is rather small. One interesting observation is that the amount of cooperation on these graphs, weighted or unweighted, is high, of the order of what has been found for unweighted BA scale-free networks [4]. The reason is simple: due to the way in which the graphs are built [20], their degree distribution turns out to be very close to scale-free (results not shown here). It is thus obvious that the games' behavior should be very similar. This in turn also shows once again that the purely topological aspects of the networks are more important than the weights in determining the steady states of the games.

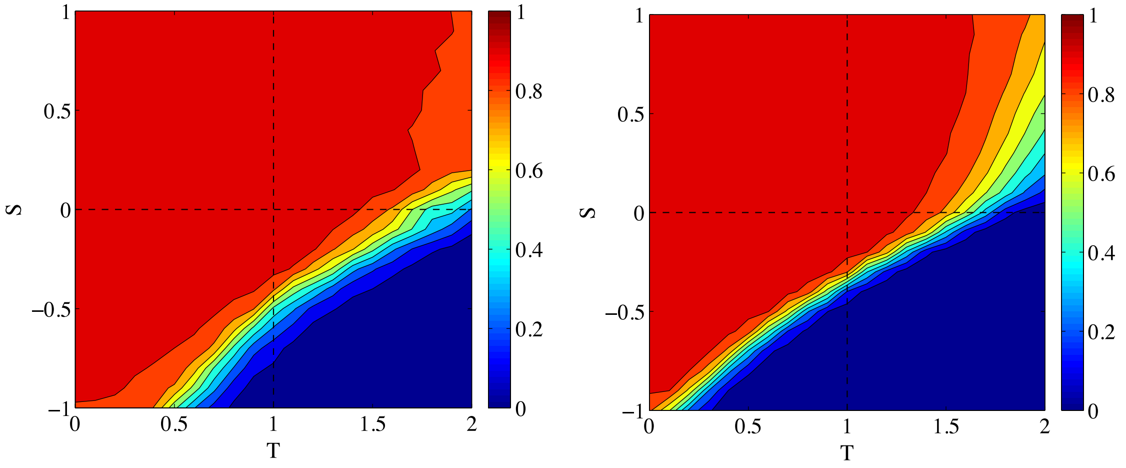

Fig. 6. Average cooperation in unweighted assembly model graphs. Left image: imitation of the best. Right image: replicator dynamics. Size of graphs is $N = 1000$. Each grid point is the average of 50 independent runs.

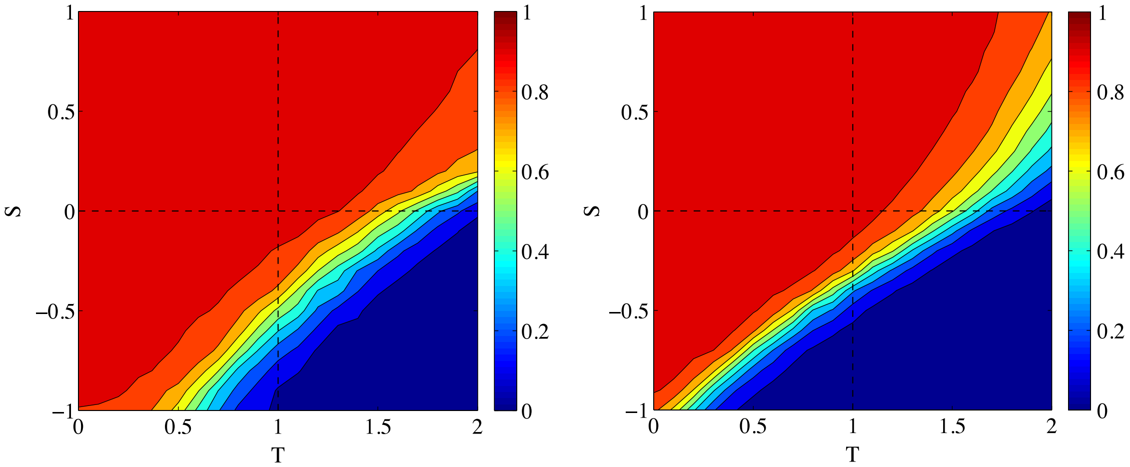

Fig. 7. Average cooperation in weighted assembly model graphs. Left image: imitation of the best. Right image: replicator dynamics. Size of graphs is $N = 1000$. Each grid point is the average of 50 independent runs.

As a further example of a weighted graph resulting from a bipartite interaction, we consider the network of scientists belonging to the Econophysics community.[2] This network has a size $N = 738$ and 1866 edges, which gives $\langle k \rangle \simeq 5.06$. In this network two nodes (authors) are connected if they have coauthored at least one scientific article. Link weights are assigned in two ways; the first corresponds to Eq. (1) in which the weight corresponds simply to the number of common papers, suitably normalized; in the second scheme, this row weight is corrected for a factor that accounts for the number of coauthors of a given paper, on the grounds that the larger the number of authors a paper has, the lesser the likelihood that the authors know each other equally well. Thus the weight of the link between two coauthors i and j is given by $w_{ij} = \sum_k \delta_i^k \delta_j^k / (n_k - 1)$ [10], where δ_j^k is 1 if author i was a coauthor of paper k and 0 otherwise, and n_k is the number of coauthors of paper k (single author papers are excluded).

Fig. 8 shows the results for the two weighting schemes described above for imitation of the best update (left figures) and local replicator dynamics update (right figures). It is apparent that results are qualitatively very similar. We can compare these results with those obtained in the unweighted graphs (Fig. 9). The behavior is similar but, on the whole, the degree of cooperation in the three non-trivial games is slightly lower for the weighted versions of this social network. Although no general conclusions can be drawn from this single instance, it can be said that, at least in this case, taking into account the strengths of ties does not help cooperation. Whether or not this is a more general phenomenon in social networks

2 Kindly provided by Zhang Peng, personal communication.

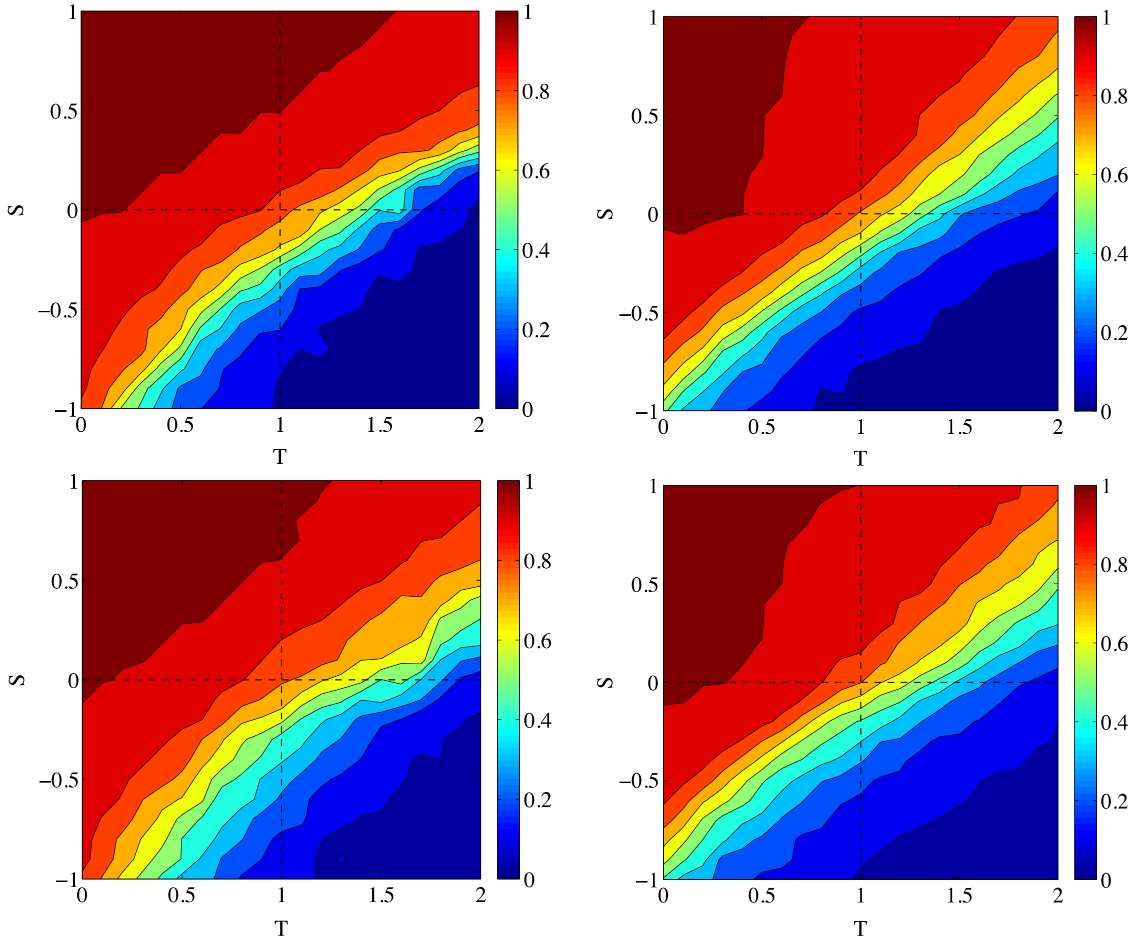

Fig. 8. Average degree of cooperation at steady state on the collaboration network of econophysicists. Left column: imitate the best. Right column: replicator dynamics. Top row: standard weighting scheme; bottom row: corrected weighting scheme (see text). Averages over 50 independent runs.

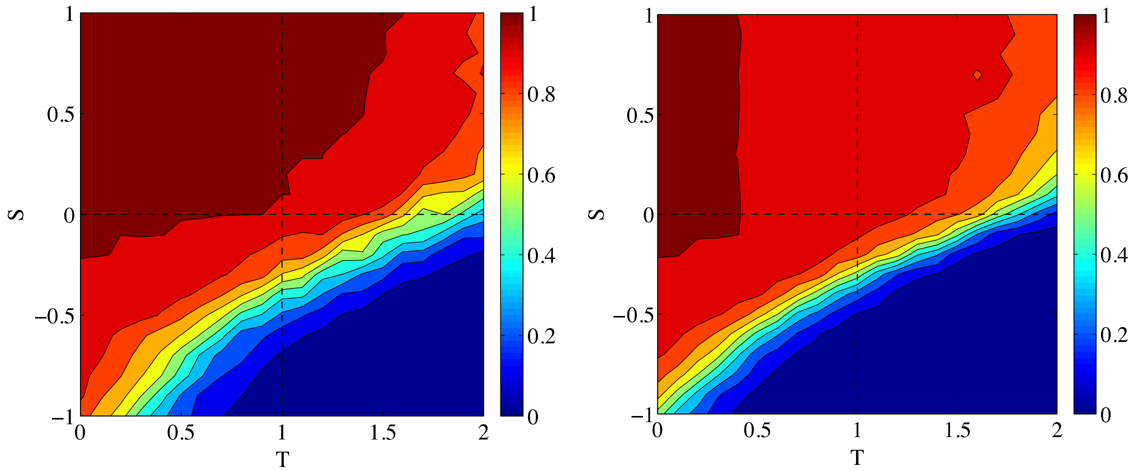

Fig. 9. Average degree of cooperation at steady state on the unweighted collaboration network of econophysicists. Left column: imitate the best. Right column: replicator dynamics. Averages over 50 independent runs.

would require a much more complete investigation using real networks coming from different types of social interactions. Unfortunately, reliable weighted network data are still few.

Table 2
Average cooperation at steady state in weighted and unweighted networks derived from bipartite graphs. 'ib' and 'rd' stand for 'imitate the best' and 'replicator dynamics' update rules respectively. PD, SG, and SH design the Prisoners Dilemma, Snowdrift, and Stag Hunt games respectively.

	PD, ib	PD, rd	SG, ib	SG, rd	SH, ib	SH, rd
Guimerà weighted	0.232	0.141	0.883	0.797	0.815	0.668
Guimerà unweighted	0.246	0.137	0.895	0.811	0.807	0.650
Econophysicists, standard weights	0.164	0.147	0.845	0.746	0.673	0.585
Econophysicists, corrected weights	0.203	0.171	0.828	0.754	0.701	0.582
Econophysicists, unweighted	0.221	0.157	0.933	0.875	0.704	0.611

Table 2 summarizes the average cooperation levels reached in the various network types studied in this section, both weighted and unweighted, for the three non-trivial games PD, SG, and SH.

4. Discussion and conclusions

The focus of this paper is on the influence of the weighted nature of social interaction networks on evolutionary games played on those networks, an issue that has been somewhat neglected until now. The first major question was: how do weighted edges affect the results of standard evolutionary games on complex networks? Owing to the lack of generally accepted theoretical models of the formation and structure of weighted social networks, we tried to answer the question by using numerical simulation and several methods for assigning weights to links. Three different roads were tried: in the first, weights were assigned to edges according to some probability distribution independently of the underlying network topology. This means that the intensity of a binary relationship does not depend on the environment of the corresponding link and goes under the name of dyadic hypothesis in social networks. In the second empirical model, the weight of a link is correlated in some way with the degrees of the end points. Finally, in the third model, we started from bipartite affiliation graphs and generated weighted graphs using the model proposed in Ref. [20] and a real collaboration graph.

The results obtained using the first uncorrelated model clearly show that the influence of weights on the games is almost negligible. Furthermore, in this case topology and weights do not interact, as shown by the results on scale-free networks. Even in the case where the network is topologically inhomogeneous, and the weights are distributed according to a power-law, there is little difference with the unweighted case.

However, available empirical data on large networks suggest that topology and degree can be correlated to some extent. Assuming the link/weight correlation found in Ref. [19] for flights between airports and used in Ref. [6] for the evolutionary PD on BA scale-free networks, which postulates that weights w_{ij} are proportional to $(k_i k_j)^\alpha$, we have numerically studied the full phase space of the standard games for several positive and negative exponent α values thus extending the work of Ref. [6] which was limited to a very small region of the game configuration space. For values of α larger than 0 and smaller than 1.5 approximately, there is indeed a non-negligible increase of the average cooperation for all the non-trivial games. However, it must be said that recent empirical research on typical social networks does not support such a weight dependence [19,8,9].

The third model comes from the realization that many social networks are of the affiliation type, which can be represented by bipartite graphs. We have thus studied model weighted networks built according to the method of Ref. [20]. On these graphs, the average results in terms of cooperation are very good but this appears to be due to the scale-free nature of the resulting networks and not to the weighted aspects, since weighted and unweighted networks give almost the same results. Finally, we examined the case of an actual coauthorship network, a projection of the bipartite graph formed by authors on the one hand, and the papers they have written together on the other. Our results show that weights are even slightly detrimental for cooperation for this particular network, although the main features remain similar with respect to the unweighted case. This last investigation was performed to illustrate the study with a real-life case but the results cannot be generalized in the absence of a sufficient amount of statistics on several social networks. In this respect, we mention that in previous work, Voelkl and Kasper studied the donor game, an analogous of the PD, using a number of networks representing weighted interactions in primate groups [7]. Using a fitness proportional update rule, they found that the fixation probability of cooperation in the groups was larger on the average with respect to a baseline well mixed population of the same size. The networks were very small and degree-inhomogeneous in many cases. Indeed, the authors refrained from attributing the results to the weighted nature of their networks; instead, they mainly invoked topological reasons and concluded that those, rather than the weights, were the more important contribution to network reciprocity. Although they did not examine the unweighted networks, in the light of the results presented here, it is likely that their explanation is essentially correct for this particular case.

Summing up all the previous considerations, a general conclusion can be drawn, taking into account the fact that our numerical simulation study cannot be considered exhaustive, but it has certainly been extensive. The conclusion is that, for well known model network classes, the weighted aspect of links does not seem to have a large influence on evolutionary games played on networks, the topological aspects being more important. It appears that weights essentially act as a source of noise on the payoff values. Thus, studying the unweighted versions of networks would seem to suffice for evolutionary games, at least in the case of standard model graphs. Of course, our conclusion does not apply to other domains. For example, link weights certainly play a very important role in diffusion and fragmentation processes on networks. Nevertheless, a thorough study of evolutionary games on reliably weighted actual social networks is still lacking. The single actual network that was studied here and the report [7] are insufficient, in our opinion, to draw any reliable conclusions. Our future work will be directed towards a better understanding of evolutionary games on real weighted networks, including dynamical ones, and the relationships with their topological features.

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